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Five hundred and ninety-second Meeting.

March 10, 1868. — ADJOURNED STATUTE MEETING.

The PRESIDENT in the chair.

The President called the attention of the Academy to the recent decease of Sir David Brewster of the Foreign Honorary Members, and of Hon. Daniel Lord, of New York, of the Associate Fellows.

Five hundred and ninety-third Meeting.

April 14, 1868. — MONTHLY MEETING.

The PRESIDENT in the chair.

The President called the attention of the Academy to the recent decease of Dr. Samuel L. Dana, of the Resident Fellows, and of Professor William Smyth, of the Associate Fellows.

The following paper was presented : —

*Dispersion of a Ray of Light refracted at any number of Plane Surfaces.** By EDWARD C. PICKERING.

LET $a_1 a_2 a_3$, &c., be the angles included between the surfaces, $n_1 n_2 n_3$ their indices of refraction, $i_1 i_2 i_3$ the angles of incidence, $r_1 r_2 r_3$ the angle of refraction ; $\sin i_1 = n_1 \sin r_1$ and in general

$$\sin i_m = r_m \sin r_m \quad (1)$$

$$\text{also} \quad i_m = a_{m-1} + r_{m-1} \quad (2)$$

As the dispersion of any portion of the spectrum is always proportional to the angular divergence of two rays of nearly equal refrangibility, if we vary $n_1 n_2$, &c., dr_m will measure the dispersion. Differentiating (1)

$$\cos i_m di_m = r_m \cos r_m dr_m + \sin r_m dn_m$$

* Since presenting this communication to the Academy, I have learned that a portion of this subject was studied by Sir David Brewster in 1812. I have, therefore, modified my paper, omitting what was not new, except when necessary to preserve the context.

if $n_1 n_2 n_3$ are all functions of N , making the latter the independent variable and dividing by dN , we have

$$\frac{dr_m}{dN} = \frac{\cos i_m}{n_m \cos r_m} \frac{di_m}{dN} - \frac{\sin r_m}{n_m \cos r_m} \frac{dn_m}{dN} \quad (3)$$

But differentiating (2)

$$di_m = dr_{m-1}$$

and calling

$$\frac{\cos i_m}{n_m \cos r_m} = a_m, \quad \frac{\sin r_m}{n_m \cos r_m} \frac{dn_m}{dN} = b_m$$

also the dispersion of a ray after passing m surfaces, or

$$\frac{dr_m}{dN} = l_m,$$

and (3) becomes

$$l_m = a_m l_{m-1} - b_m \quad (4)$$

This formula by successive substitutions may be applied to any case. For a single surface

$$m = 1, l_0 = 0,$$

and hence

$$l_1 = a_1 0 - b_1 = -\frac{1}{n_1} \tan r_1$$

which equals unity when $\tan r_1 = n_1$ or at the angle of total polarization. That is, the unit of dispersion is that produced by a single surface when the ray is in the position of total polarization. For two surfaces (4) becomes

$$l_2 = a_2 l_1 - b_2 = -(a_2 b_1 + b_2);$$

in a prism $n_2 = \frac{1}{n_1}$ making suitable substitutions, and reducing we obtain

$$l_2 = \frac{\sin a}{\cos r_1 \cos r_2}$$

a being the angle of the prism. For minimum deviation

$$r_1 = \frac{a}{2}, \quad r_2 = i_1 \text{ and } l_2 = \frac{2}{n} \tan i;$$

this does not, however, give the minimum of dispersion for which the condition (found by differentiating) is

$$n^2 \sin (a + r_1) \cos (a + 2 r_1) + \sin r_1 = 0 \quad (5)$$

an equation which has, I believe, never been solved. When a is very small

$$\sin (a + r_1) = a + r_1, \cos (a + 2 r_1) = 1 \sin r_1 = r_1$$

since r_1 is small also, and (5) becomes

$$n^2 (a + r_1) + r_1 = 0; r_1 = a \frac{n_2}{n_2 + 1}.$$

I find that this equation gives very nearly the true value of r_1 , even when a is large, thus for $n = 1.5$, $a = 15^\circ, 30^\circ, 45^\circ$, and 60° it gives $r_1 = 10^\circ 23', 20^\circ 46', 31^\circ 9'$, and $41^\circ 32'$, while the true values obtained by trial in (5) were $10^\circ 23', 20^\circ 45', 31^\circ 6'$ and $41^\circ 29'$.

To show the comparative deviation and dispersion, while a prism is rotated, I have calculated the following table for prisms whose index of refraction is 1.5, and their angle $15^\circ, 30^\circ, 45^\circ$, and 60° ; r_2 is the angle at which the light leaves the prism, and i_1 that at which it enters. The columns headed "Reduced Deviation" are calculated by the condition that for small angles at which the dispersion and deviation are proportional their units shall be the same.

From this table it follows that if we turn the prism of a spectroscope 20° from minimum deviation, the dispersion will be nearly doubled, thus doubling the power of the instrument. In projecting the spectrum on the screen, this device is often useful; I have thus, by a single 60° prism, projected a spectrum of such a size that the two parts of the D line were about a millimetre and a half apart, each having a thickness of only about half a millimetre.

Since the dispersion does not depend strictly upon the deviation, and we may have two prisms producing the same dispersion but unequal deviations, we can evidently make one achromatize the other, even if they are made of the same kind of glass, and have the same angle. Thus if two 15° prisms be so placed that r_2 shall be 50° for one, and $37^\circ 21'$ for the other, the dispersions of each will be .418, while the deviation will be $11^\circ 7'$ for one, and $17^\circ 53'$ for the other, as may be seen from the table. Again, if the ray of light passes through first one and

15° Prisms.									
r_2	i_1	Devia- tion.	Reduced devia- tion.	Disper- sion.	r_2	i_1	Devia- tion.	Reduced devia- tion.	Disper- sion.
—42 35	90 0	32 25	1.131	.472	22 51	0 0	7 51	.272	.281
—40 0	76 18	21 18	.743	.443	30 0	6 42	8 18	.289	.300
—30 0	58 6	13 6	.457	.362	40 0	15 28	9 32	.332	.344
—20 0	45 6	10 6	.352	.312	50 0	23 53	11 7	.388	.418
—10 0	33 38	8 38	.302	.283	60 0	31 18	13 42	.478	.552
0 0	22 51	7 51	.274	.268	70 0	37 13	17 47	.616	.827
6 56 α	15 41	7 37	.265	.265	80 0	41 11	23 49	.831	1.658
10 0	12 35	7 35	.264	.265	85 0	42 13	27 47	.970	3.354
11 17 β	11 17	7 34	.264	.266	90 0	42 35	32 35	1.131	∞
20 0	2 44	7 44	.270	.276					
30° Prisms.									
—17 53	90 0	42 7	1.470	.705	40 0	6 57	16 57	.592	.655
—10 0	63 34	23 34	.822	.633	48 35	0 0	18 35	.648	.756
0 0	48 35	18 35	.648	.577	50 0	1 3	18 57	.655	.778
10 0	36 30	16 30	.576	.559	60 0	7 55	22 5	.761	1.000
13 56 α	32 8	16 4	.561	.551	70 0	13 15	26 45	.934	1.480
20 0	25 43	15 43	.549	.556	80 0	16 42	34 18	1.197	2.934
22 51 β	22 51	15 42	.548	.562	85 0	17 35	37 25	1.306	5.857
30 0	15 55	15 55	.556	.587	90 0	17 53	42 7	1.470	∞
45° Prisms.									
0 0	imag.			1.000	40 0	30 16	25 16	.882	.980
4 47	90 0	49 47	1.738	.952	50 0	21 45	26 45	.934	1.135
10 0	68 33	33 33	1.171	.919	60 0	14 40	29 40	1.036	1.435
20 0	52 16	27 16	.952	.885	70 0	9 20	34 20	1.198	2.080
21 7 α	50 47	26 54	.938	.885	80 0	5 57	40 57	1.429	4.080
30 0	40 17	25 17	.882	.905	85 0	5 5	45 5	1.571	8.127
35 2 β	35 2	25 4	.875	.934	90 0	4 47	49 47	1.738	∞
60° Prisms.									
27 55	90 0	57 55	2.022	1.315	50 0	47 14	37 14	1.300	1.545
28 27 α	83 35	52 2	1.816	1.315	60 0	38 52	38 52	1.356	1.907
30 0	77 7	47 7	1.645	1.317	70 0	32 53	42 53	1.496	2.716
40 0	58 29	38 29	1.343	1.374	80 0	29 12	49 12	1.717	5.273
48 35 β	48 35	37 10	1.297	1.511	90 0	27 55	57 55	2.022	∞

 α Minimum of Dispersion. β Minimum of Deviation.

then the other, the second will neutralize the dispersion of the first, while there will remain $17^\circ 53' - 11^\circ 7' = 6^\circ 46'$ deviation. It was by this arrangement of prisms that Brewster obtained achromatism with one kind of glass.

Perhaps the most important application of these principles is to photographing the spectrum. Photographs taken by the common methods are greatly distorted, particularly in the more refrangible end. They

can only be used by identifying the more prominent lines, with those whose wave-length is known, and interpolating the remainder approximately. If, as is often the case, these standard lines cannot be recognized, the photograph becomes useless. To show the amount of distortion, suppose a spectrum to contain three similar double lines *A*, *B*, and *C*, whose indices of refraction are 1.5, 1.6, and 1.7, and that we use a 60° prism, the line *B* being in the position of minimum deviation. The deviations of the three lines will then be 48° 34', 53° 8', and 58° 11', and their dispersion 1.528, 1.667, 1.878; that is, *A* and *C* will be at distances of 4° 34' and 5° 3' from *B* instead of equidistant, and the interval between the components of each line will be as 1.528 : 1.667 : 1.878; the distortion in this case amounting to about 20 per cent.

If now the portion of the screen which receives the line *C* be brought nearer the prism, the parts of this line will approach one another, and since their distance apart is proportional to their distance from the prism, the three lines will appear alike, if the screen is so inclined that the points where they are projected are at distances,

$$\text{as} \quad \frac{1}{1.528} : \frac{1}{1.667} : \frac{1}{1.878}$$

$$\text{or as} \quad .654 : .600 : .532.$$

A simple calculation shows that the screen must be slightly curved to fulfil this condition, but if plane, the distortion will be only about one and a half, instead of twenty per cent, the angle of inclination with the ray *B* being about 37°. If an achromatic lens was used for the projection, all parts of the spectrum would not be in focus, but with a single lens the focal distance of the violet is always less than that of the red rays. If then we use such a lens, inclining the screen at the same time corrects the distortion, and brings all parts into focus at once. By placing the prism at a suitable distance from the lens, both sources of error may be almost entirely eliminated. The oblique incidence of the light on the sensitive surface would be an objection to this method, but would be partly counterbalanced by the fact that the length of the spectrum would be thereby increased more than one half. Or, if preferred, the prism may be turned, so that, applying the correction for distortion as above, the screen shall be more nearly perpendicular to the light.

In conclusion, this spectrum would possess the following advantages

over the distorted forms now in use. Horizontal distances being proportional to the change in the index of refraction, the latter could be determined at once for any line, by a scale of equal parts. Its extent would be much greater than that of the visible spectrum, and we could determine the index of refraction of rays of too short wave-length to be measured readily by the common methods. It would be a normal spectrum for any given material, being independent of the form and position of the prism. And (especially if the interference bands were produced in it) it would afford, from its extent, great advantages for the study of the laws of the dispersion of light by different substances.

Five hundred and ninety-fourth Meeting.

May 12, 1868. — MONTHLY MEETING.

The PRESIDENT in the chair.

The Corresponding Secretary read letters relative to exchanges ; also a letter from Professor De La Rive in acknowledgment of his election into the Academy as a Foreign Honorary Member.

Mr. C. M. Warren presented by title a memoir on "Volatile Hydrocarbons in Pennsylvania Petroleum."

DONATIONS TO THE LIBRARY,

FROM JUNE 2, 1865, TO JUNE 30, 1866.

State of Massachusetts.

Report to His Excellency the Governor and the Honorable Council, of the Commissioners appointed under the Resolve of May 3, 1865, "concerning the Obstructions to the Passage of Fish in the Connecticut and Merrimack Rivers." 8vo pamph. Boston. 1866.

Massachusetts Historical Society.

Proceedings. 1864-1865. 8vo. Boston. 1866.

Massachusetts Institute of Technology.

First Annual Catalogue of the Officers and Students, and the Programme of the Course of Instruction, of the School of the Massachusetts Institute of Technology. 1865-66. 8vo pamph. Boston. 1865.